Area of a triangle

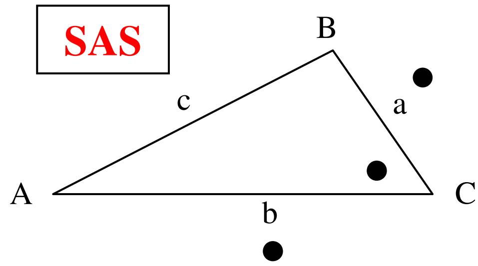
Recall the formula for the area of a triangle. $\text{Area} = \frac{1}{2}\,\text{base} \times \text{height}$

However, sometimes we do not have the height.

We may have **two sides** and the **angle between** them.

SAS

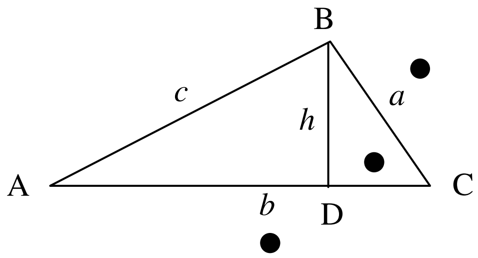
We will develop a new formula for this case.



$$\boxed{\text{Area} = \frac{1}{2}\,a\,b\,\sin C}$$

THEORY

Assume we have sides a, b and angle C.



Drop a perpendicular from B to AC meeting AC at D
and let the length of this line be h.

Using our original formula for the Area

we get

$\text{Area} = \frac{1}{2}\,\text{base} \times \text{height}$ $\qquad$ $\text{Area} = \frac{1}{2}\,b \times h \quad \ldots \text{(i)}$

However, we do not have h.

But, triangle BDC is right angled and so, $\sin C = \frac{h}{a}$

Rearranging this we get: $h = a \sin C$

and substituting in (i) gives $\qquad \text{Area} = \frac{1}{2}\,b \times a \sin C$

Which is usually written as

$$\text{Area} = \frac{1}{2}\,a\,b\,\sin C$$

This can also be cyclically permuted.

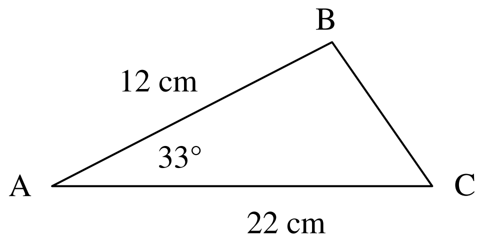
Remember as:

half the two sides multiplied together × the sine of the angle between them

Area of a triangle

Example

Find the area of triangle ABC



Using the formula: $Area = \frac{1}{2}ab\sin C$

or remembering

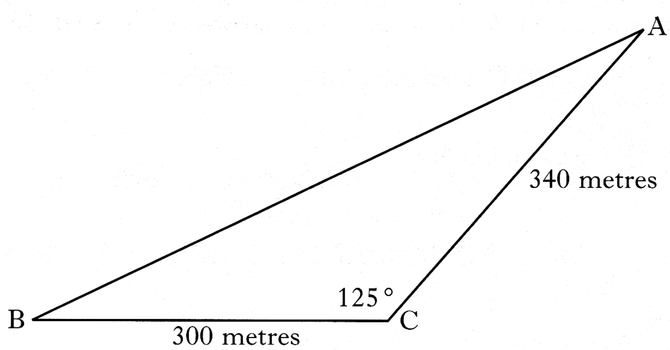
half the two sides multiplied together × the sine of the angle between them.

or cyclically permute.

$Area = \frac{1}{2} \times 12 \times 22 \times \sin 33°$ Area = 71.9 cm^2

Past Paper Questions:

1. A field, ABC, is shown in the diagram.



Find the area of the field.

[Ans. = 41776.8 m^2]

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2. Calculate the area of the triangle.

[Ans. = 34.9 cm^2]

[Hint: find the 3rd angle]